A Review Talk on Diff, Weyl and Conf

Daniel Martin
Durham University
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Outline

1. Diff, Weyl and Conf
2. Dilatations as RG
3. Anomalies
4. Example Theories

Diffeomorphisms

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We can generate a family of infinitesimal diffs from $\mathcal{M}$ to itself using a continuous vector field ξ^μ.

In this case, if co-ordinates have been laid down, the point at x^μ maps to the point at

$$x'^\mu(x) = x^\mu + \epsilon\, \xi^\mu(x) + \mathcal{O}(\epsilon^2).$$

Variations

A field ϕ_i has *variations*, defined by a one-parameter family of field states $\phi(x;\lambda)$:

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This induces a variation in the Lagrangian:

$$\delta\mathcal{L} = \frac{\delta\mathcal{L}}{\delta\phi}\delta\phi \equiv \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial_\mu(\delta\phi) + ...$$

δ is a ‘symmetry’ when $\delta S = 0$.

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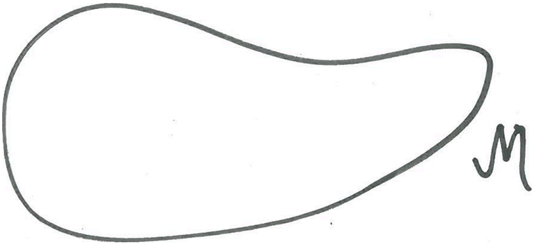
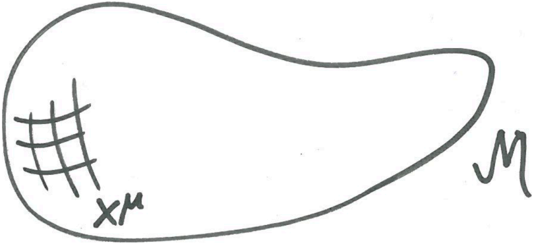
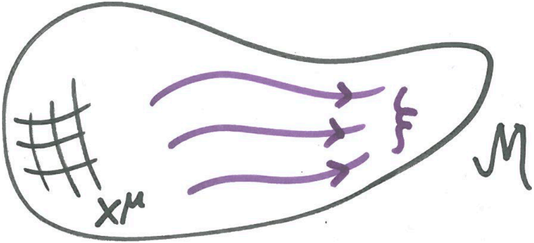
In particular, $\delta g_{\mu\nu} = \pounds_\xi g_{\mu\nu}$.





x^μ
$\mathcal{M}$



x^μ
ξ
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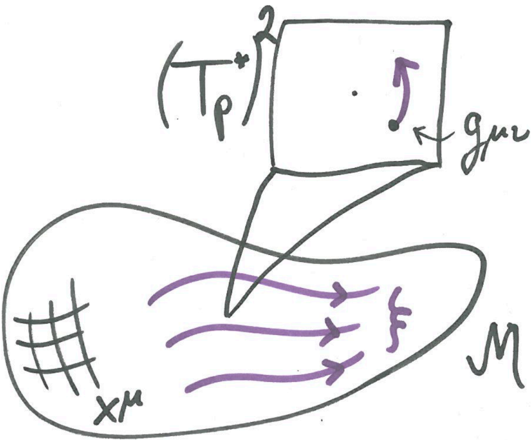
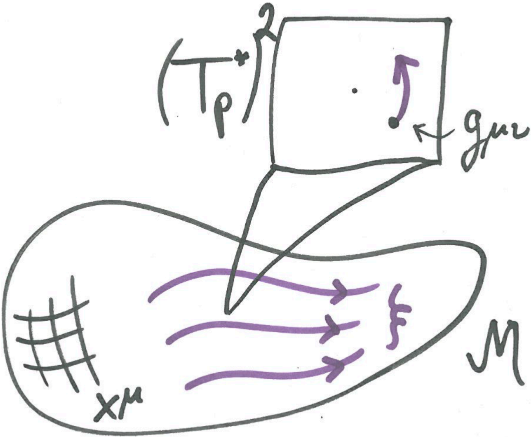


$(T_p^*)^2$
$g_{\mu\nu}$
ξ
x^μ
M



In this hour we shall not change co-ordinates a single time!!

Weyl Transformations

Weyl transformations are variations

$$\begin{aligned} \delta g_{\mu\nu} &= 2\,\lambda(x)\,g_{\mu\nu}\,, \\ \delta\phi_i &= -\Delta_i\,\lambda(x)\,\phi_i \end{aligned}$$

where Δ_i is the *scaling dimension* of the field ϕ_i.

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With diff $+$ Weyl you can set 2d metrics to anything (smooth) unless there is a topological obstruction.

A simple diff$\times$Weyl-invariant theory

A simple diff×Weyl-invariant theory

'Conformally coupled' scalar in 4d:

$$\mathcal{L} = \sqrt{-g}\Big(\frac{1}{2}\nabla^{\mu}\varphi\nabla_{\mu}\varphi + \frac{1}{12}R\varphi^{2} - \frac{\lambda}{4}\varphi^{4}\Big).$$

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One can use Weyl symmetry to gauge-fix $\varphi = v$:

$$\mathcal{L} = \sqrt{-g}\left[\frac{Rv^2}{12} - \frac{\lambda v^4}{4}\right].$$

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- The Weyl symmetry has been fully gauge-fixed.
- The residual gauge symmetry is diff.

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- The symmetry that remains is the subgroup of diff $\times$ Weyl for which $\delta g^B_{\mu\nu} = 0$. These are the conformal transformations!

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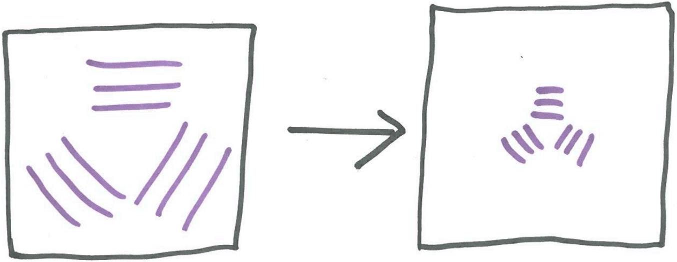
- There is a difference between a gauge-fixed dynamical metric and a non-dynamical metric. In the former case, the presence of a quantum Weyl anomaly would break a gauge redundancy, whereas in the latter case a Weyl anomaly is fine.

Conformal Transformations

A conformal transformation in the sense of CFT changes the physical distance between physical objects in the theory.



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The effect on the dynamical fields is

$$\delta \phi_i = \pounds_\xi \phi_i - \Delta_i \left(\frac{1}{d} \partial_\mu \xi^\mu \right) \phi_i .$$

Non-Trivial Topologies

This discussion was entirely at the local level.

Non-trivial topologies will make the symmetries much richer and less trivial.

RG in Quantum Field Theory

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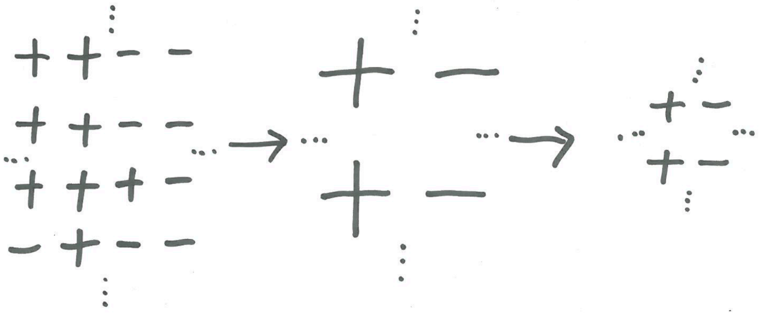
Classical fields ϕ_i are replaced by local operators $\mathcal{O}_i$. A set of variations $\delta\mathcal{O}_i$ are now arbitrary (local) operators rather than arbitrary c-numbers.

RG in Quantum Field Theory

RG transformation = integrate out degrees of freedom + dilatation.



Dilatations as RG

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Generally dilatations will not be a symmetry and instead

$$\delta_{\text{Dilatation}} \langle O_i \rangle = \delta_{\text{Weyl}} \langle O_i \rangle = \langle \delta_{\text{Weyl}} O_i \rangle + \int d^d x \left\langle \lambda(x) T^{\mu}{}_{\mu}(x) O_i \right\rangle$$

in which we used that path-integrals are coordinate-invariant, and the definition of the quantum $T_{\mu\nu}$ operator

$$\frac{\delta \langle ... \rangle}{\delta g_{\mu\nu}} = \int d^d x \frac{\sqrt{-g}}{2} \langle T^{\mu\nu}(x) \, ... \rangle \,.$$

Dilatations as RG

Define a Weyl scaling in a quantum theory as

$$\begin{aligned} \delta g_{\mu\nu} &= 2\,\lambda(x)\,g_{\mu\nu}, \\ \delta \mathcal{O}_i &= -\lambda(x) \sum_j \Delta_i^{\ j}\,\mathcal{O}_j \end{aligned}$$

where we have allowed for the possibility that local operators mix into each other. Our transformation has $\lambda = 1$.

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Also, expand $T^{\mu}_{\;\;\mu}$ in terms of a complete basis of local operators:

$$\begin{aligned} \int d^d x\; T^{\mu}_{\;\;\mu} &= \sum_k \int d^d x\; \beta^k(g) \mathcal{O}_k \\ &= \sum_k \beta^k(g) \frac{\partial}{\partial g^k} S \end{aligned}$$

for $S = \int d^d x\; \sum_i g^i \mathcal{O}_i(x)$.

Dilatations as RG

This leaves

$$\delta_{\text{Weyl}} \langle O_i \rangle = -\sum_j \Delta_i^{\ j} \langle \mathcal{O}_j \rangle - \sum_k \beta^k(g) \frac{\partial}{\partial g^k} \langle O_i \rangle \,.$$

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Thus a dilatation is equivalent to a change in the couplings plus a mixing of operators. The Weyl transformation has also moved the cutoff, so finally we have the Callan-Symanzik equation:

$$\left[\mu \frac{\partial}{\partial \mu} + \sum_k \beta^k(g) \frac{\partial}{\partial g^k} \right] \langle O_i \rangle + \sum_j \Delta_i^{\ j} \langle \mathcal{O}_j \rangle = 0.$$

Curved Space RG

What is curved space RG flow?

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$$\partial_\mu j^\mu = -\frac{ie^2}{24\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

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If A_μ were a non-dynamical background field, this would not spoil the consistency of the theory.

Anomalies

However when A_μ is a gauge field, we have a problem. We know that

$$\partial_\mu J^\mu = \delta_{\text{gauge}}\, \Gamma[A_\mu]$$

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Thus a physical quantity is no longer gauge invariant if the current is not conserved. The quantum theory does not exist.

Weyl Anomaly

Classically Weyl-invariant theories can have anomalies.
[Capper, Duff, 1974]

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- One easy way to get this is by integrating up

$$\delta_{\text{Weyl}} \left\langle T^{\mu}{}_{\mu}(x) \right\rangle = \int d^2y \left\langle T^{\mu}{}_{\mu}(x)\, \lambda(y)\, T^{\nu}{}_{\nu}(y) \right\rangle .$$

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Hence in 2d

$$\text{conformal symmetry} \xrightarrow{\text{anomalously broken}} SL(2,\mathbb{C}).$$

diff Anomaly

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In 2d conformal theories a condition for vanishing diff-anomaly is $c = \tilde{c}$.

Examples: QCD

QCD with massless quarks:

$$\left\langle T^{\mu}{}_{\mu} \right\rangle = \frac{\beta(g)}{2g^3} \underbrace{\left\langle G^a_{\mu\nu} G_{a\mu\nu} \right\rangle}_{\sim \mathcal{O}(\Lambda^4_{\text{QCD}})}.$$

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However if RG flow eventually takes $\beta(g) \to 0$ and $\alpha_s \to$ finite, we are in a phase of matter described by an interacting conformal field theory at long distances.

This indeed happens for $SU(3)$ as N_f is raised above 3, but it is not known precisely when.

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In differential-geometric form notation,

$$S_{CS} = \int A \wedge dA$$

does not depend on $g_{\mu\nu}$, and can be defined on manifolds prior to a metric structure.

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This is one way of seeing that Chern-Simons has no local degrees of freedom at all.

(However, being a topological theory, it induces degrees of freedom on the boundary of the manifold it is placed on.)

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In fact, in terms of e and ω (the vielbein and spin connection),

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$$S_{\text{2+1-dim GR}} = S_{\text{CS}},$$

where the Chern-Simons gauge field has the gauge group $SO(2,2)$ and is built out of e and ω.

The infinitesimal symmetries are related by

$$\delta_{\text{diff}} \Longleftrightarrow \delta_{\text{gauge}}.$$

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The connection between the two theories is not entirely clear: for example, the solution $A = 0$ in Chern-Simons would to correspond to $e = \omega = 0$, which makes no sense.

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The connection between the two theories is not entirely clear: for example, the solution $A = 0$ in Chern-Simons would to correspond to $e = \omega = 0$, which makes no sense.

However this has been a fruitful starting point for an attempt to find a consistent quantum theory of GR.

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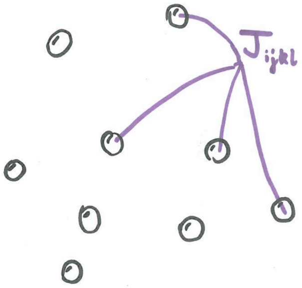
$$H = \frac{1}{4} \sum_{i,j,k,l=1}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$



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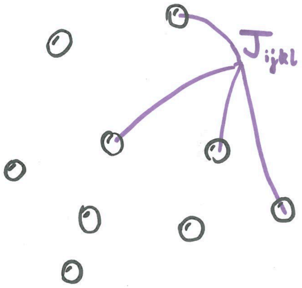

Each χ_i is a self-adjoint fermionic operator on the ith site, each J_{ijkl} is a Gaussian random variable, and $N \gg 1$.

Remarkably this model is solveable, despite becoming strongly coupled in the infrared.

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This propagator is invariant only under the $SL(2, \mathbb{R})$ subgroup of conformal symmetry, so the vacuum of the theory spontaneously breaks the symmetry to $SL(2, \mathbb{R})$.

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For example, the 4-pt function can be written as a sum over an infinite tower of conformal blocks. SYK is a member of a new class of conformal theories.

However, one ‘conformal block’ does not transform conformally, and this block actually dominates at large times. There is a non-conformal mode propagating in the IR. SYK flows to a ‘nearly conformal’ theory.

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A wide variety of quantum field theories, beyond just supersymmetric ones, admit a dual description in gravity. Many 1+1 CFT's are dual to 2+1 gravity.

One might imagine the simplest model of AdS/CFT would be

$$0+1 \text{ CFT} \Longleftrightarrow 1+1 \text{ gravity in AdS.}$$

Degree-of-freedom counting claims 1+1 gravity has -1 propagating degrees of freedom, but this can be regulated by adding a dilaton.

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The dynamics of such theories are dominated by the dilaton, which fluctuate the metric away from AdS to an $SL(2,\mathbb{R})$-invariant ‘nearly AdS’.

The action for the dilaton dynamics is the same as for the mode which breaks conformal symmetry in SYK.

This similarity to the SYK model would be explained if SYK realises some essential features of gravity.

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- Manifest invariance under reparatermizations can be realised in many interesting ways.